INFLUENCE OF NOISE ON THE SELF-POLARIZATION EFFECT OF IMPURITY DOPED QUANTUM DOTS

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Abstract:

Present study inspects the profiles of self-polarization effect (SPE) of impurity doped GaAs quantum dots (QDs) in presence of noise. Noise term maintains a Gaussian white character and it has been introduced to the system via two different pathways; additive and multiplicative. In view of a comprehensive analysis, modulation of SPE has been scrutinized along with the variations of several relevant quantities such as electric field, magnetic field, confinement potential, dopant location, dopant potential, noise strength and aluminium concentration (only for $Al_xGa_{1-x}As$ QD). Application of noise affects SPE noticeably. However, the extent to which SPE is altered depends on the noise strength domain and the pathway through which noise is applied. The outcomes of the study delineate viable routes to tune the SPE of doped QD system, particularly in presence of noise.

Keywords: quantum dot; impurity; electric field; self-polarization effect; Gaussian white noise.

1. Introduction

Low-dimensional semiconductor systems (LDSS) such as quantum wells (QWLs), quantum wires (QWRs) and quantum dots (QDs) are characterized by immense quantum confinement much more than their bulk relatives. Such extreme confinement leaves its signature through the emergence of partial or complete quantization of electrons in LDSS to a discrete spectrum of energy levels. Moreover, such stringent confinement has helped the LDSS to come out as prolific candidates to be used for the manufacture of high-performance optoelectronic devices with splendid optical and electrical characteristics. Besides its technological importance, study of LDSS also possesses a pedagogical need for understanding and appreciating many fundamental physical concepts. As an obvious outcome we envisage sufficient number of studies on the nonlinear optical (NLO) properties of LDSS.

The probability distribution of electrons and accordingly the spatial orientation of energy levels are severely affected by the presence of impurities. The truncated spatial freedom in LDSS drives the electrons reside a considerable amount of time in proximity of impurity. Such proximity results into strong mutual interaction between them reflected through enhanced binding energy (BE) of the system. Thus, incorporation of impurity can cause discernible changes in the thermal, transport, optical and electrical properties of LDSS, principally at low temperatures. These interesting features have motivated a large number of studies on various properties of LDSS containing impurity [1–34].

Applied electric field can cause remarkable alterations in LDSS as it causes polarization of carrier distribution, energy shift of quantum states and modification of effective well width. As a result of change in the well width, effective confinement of the system also changes leading to changed energy spectrum. Such changed energy spectrum of the confined states of the carriers, in turn, affects the electronic and optical properties of LDSS with promising scope of fabricating elegant optoelectronic devices with improved NLO properties.

Exploration of *polarizability* and *self-polarization effect (SPE)* of LDSS merits importance as it provides important information about the carrier dynamics and NLO properties of them. Thus, there exists a hefty amount of studies concerning polarizability [35–56] and SPE [57–64] of LDSS. SPE is defined as the influence of well-potential on impurity. In this case the electronic wave function is affected by impurity and well potential together. The well potential (confinement potential) shifts the electronic probability distribution with respect to impurity location. The polarization thus generated is called SPE.

Introduction of *noise* to LDSS can noticeably alter its performance. Noise may originate externally, or it may be intrinsic, emerging from the changes in the structure of QD lattice in the vicinity of impurity. It is therefore quite relevant to explore how presence of noise perturbs various properties of impurity doped LDSS. In the present work we examine the *SPE* of *GaAs* QD doped with impurity in presence of noise following the variations of several important quantities such as electric field (F), magnetic field (B), confinement potential (ω_0), dopant location (r_0), dopant potential (V_0) and noise strength (ζ). Furthermore, we have also explored $Al_xGa_{1-x}As$ QD in order to understand the role played by aluminium concentration (x) in shaping SPE. Present study considers a 2-d QD (*GaAs*) carrying a single electron in presence of a static electric field. The confinement is parabolic in the $x-y$ plane. An orthogonal magnetic field is present too as an additional confinement. Impurity, modeled by a Gaussian potential, has been doped into the QD system. Gaussian white noise has been externally applied to the system which initiates substantial disorder.

There are two different pathways (modes) through which such introduction of disorder can be achieved. These two modes are additive and multiplicative which differ from one another by the extent of interaction with the system. The investigation elucidates how delicately noise (which evidently depends on its mode of application) modulates the SPE of doped QD when several important parameters are varied over a range.

2. METHOD

The system Hamiltonian with impurity (H_0) contains four terms and reads as:

$$H_0 = H_0' + V_{imp} + |e| F\left(x+y\right) + V_{noise} \qquad (1)$$

In the above expression, the first, second, third and the fourth terms on the right hand side of the equation stand for impurity-free system containing single carrier electron, the impurity potential, the externally applied electric field having field strength F and noise contribution, respectively. The static electric field has been applied along x and y-directions. $|e|$ is the absolute value of electron charge. The noise term characterizes zero mean and spatially δ-correlated Gaussian white noise (additive/ multiplicative).

In view of a lateral parabolic confinement in the $x-y$ plane and presence of a perpendicular magnetic field, H_0', under effective mass approximation, can be written as:

$$H_0' = \frac{1}{2m^*}\left[-i\hbar\nabla + \frac{e}{c}A\right]^2 + \frac{1}{2}m^*\omega_0^2\left(x^2+y^2\right), \qquad (2)$$

where m^* is the effective mass of the electron in QD and ω_0 is the harmonic confinement frequency. **A** is the vector potential which in Landau gauge becomes $A = (By, 0, 0)$, where B is the magnetic field strength. In this gauge H_0' can be further written as:

$$H_0' = -\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{1}{2}m^*\omega_0^2 x^2 + \frac{1}{2}m^*\Omega^2 y^2 - i\hbar\omega_c y\frac{\partial}{\partial x} \qquad (3)$$

where the quantity $\Omega\left(=\sqrt{\omega_0^2+\omega_c^2}\right)$ represents the effective confinement frequency in the y-direction, $\omega_c\left(=\frac{eB}{m^*c}\right)$ being the cyclotron frequency.

V_{imp} represents the Gaussian impurity (dopant) potential and can be expressed as $V_{imp} = V_0 e^{-\gamma\left[(x-x_0)^2+(y-y_0)^2\right]}$. The relevant parameters belonging to this dopant potential are (x_0, y_0), V_0 and $\gamma^{-1/2}$. They represent the site of dopant incorporation, magnitude of the dopant potential, and the spatial region over which the impurity potential is dispersed, respectively. γ can be given by $\gamma = k\varepsilon$, where k is a constant and ε is the static dielectric constant of the medium. The noise term of eqn.(1) can be generated by Box-Muller algorithm with necessary characteristics as mentioned before. The interaction of noise with system can be tuned in two distinct modes (pathways); additive and multiplicative. These two modes actually signify varied extents of system-noise interaction. The time-independent Schrödinger equation has been solved numerically by diagonalizing the Hamiltonian matrix (H_0). The said matrix has been generated by the direct product basis of the harmonic oscillator eigenfunctions. The necessary convergence test has been performed and finally we have obtained the energy levels and wave functions.

SPE of impurity doped QD can be given by:

$$\frac{P}{e} = -\langle\psi|(x-x_0)|\psi\rangle + \langle\psi'|(x-x_0)|\psi'\rangle - \langle\psi|(y-y_0)|\psi\rangle + \langle\psi'|(y-y_0)|\psi'\rangle \tag{4}$$

where ψ is the wave function describing the system and ψ' is the wave function in absence of confinement effects.

3. Results and Discussion

The calculations are performed using ε=12.4, m^*=0.067m_0, where m_0 is the free electron mass. The values of the other parameters (when they are kept fixed) are: $\hbar\omega_0$ =250.0 meV, F=100 kV/cm, B = 20.0 T, V_0 = 280.0 meV, r_0 = 0.0 nm and ζ = 1.0 × 10^{-2}. The parameters are suitable for *GaAs* QDs.

3.1. Role of electric field (*F*)

Figure 1 depicts the profile of SPE as a function of electric field strength (*F*) without presence of noise [Figure 1(i)] and in presence of additive [Figure 1(ii)] and multiplicative [Figure 1(iii)] noise, respectively.

SPE exhibits prominent enhancement with *F* in absence of noise [57–61, 63] up to $F \sim 280$ kV/cm beyond which it saturates with further increase in *F*.

The behavior indicates significant elongation of the extension of electronic wave function with increase in *F*. However, beyond $F \sim 280$ kV/cm the elongation settles to a steady extent and shows indifference to electric field strength. Thus, dependence of SPE on *F* becomes prominent at low *F* and at large *F* the dependence weakens. This is because of the fact that at low *F* energy levels do not shift appreciably and SPE can be distinctly noticed. In presence of additive and multiplicative noise SPE exhibits similar behavior as in absence of noise; but with much reduced magnitude. The observed behavior suggests that presence of noise causes drastic reduction in the SPE value from that of noise-free condition.

In other words, incorporation of noise induces substantial quenching of spatial extension of wave function. It needs to be mentioned now that the reduction in the values of SPE in presence of noise has been observed in all the cases where different parameters are varied over a range. Therefore, we would not mention it henceforth in the ensuing discussions.

3.2. Role of magnetic field (*B*):

Figure 2 shows the variation of SPE as a function of magnetic field strength (*B*) in absence of noise [Figure 2(i)] and under applied additive [Figure 2(ii)] and multiplicative [Figure 2(iii)] noise, respectively.

Regardless of presence of noise SPE decreases prominently with increase in *B* [60]. The observed behavior indicates steady enhancement of confinement with increase in *B* which reduces the spatial extension of wave function and SPE decreases.

3.3. Role of confinement potential (ω_0):

Figure 3 shows the variation of SPE as a function of $\hbar\omega_0$ in absence of noise [Figure 3(i)] and under applied additive [Figure 3(ii)] and multiplicative [Figure 3(iii)] noise, respectively. Both in absence and presence of noise SPE undergoes a steady decrease as ω_0 increases [57, 63] up to $\hbar\omega_0$ ~ 260 meV. SPE displays steady behavior as soon as $\hbar\omega_0$ exceeds above mentioned values. The observations further indicate persistent decrease in the spatial dissemination of the wave function with increase in the confinement potential. And there is a typical value of this confinement potential beyond which no further shrinkage of spatial spread of wave function can take place.

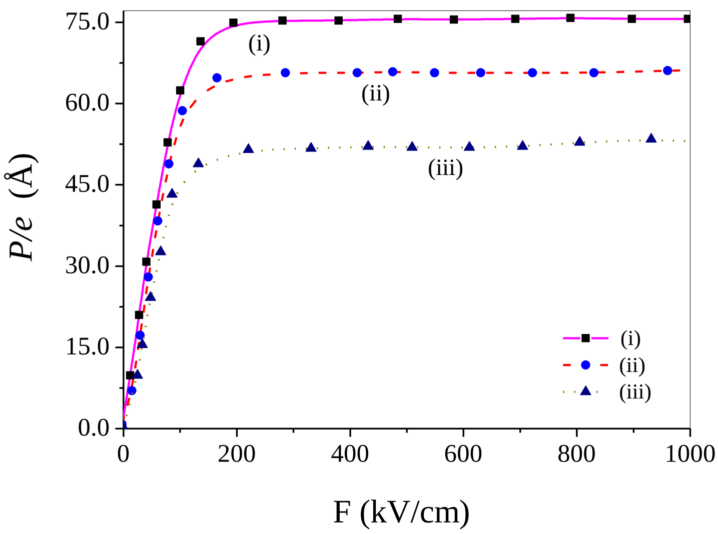

Figure 1: Plots of SPE vs *F*: (i) without noise, (ii) noise applied in additive mode and (iii) noise applied in multiplicative mode.

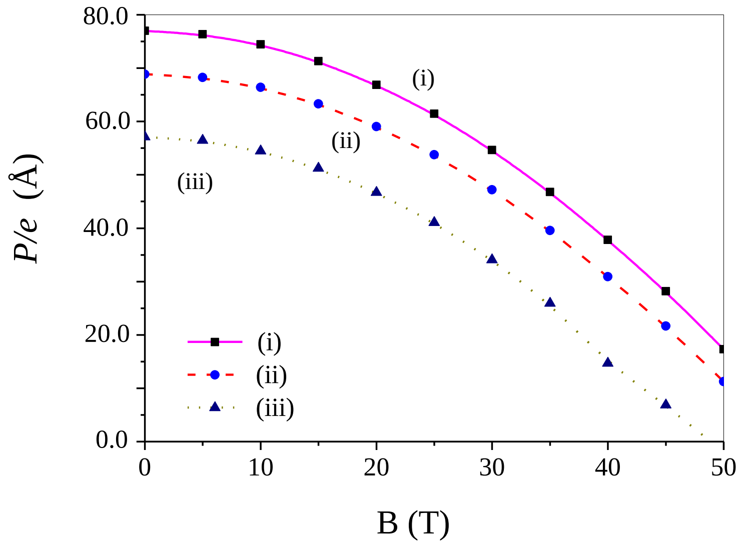

Figure 2: Plots of SPE vs *B*: (i) without noise, (ii) noise applied in additive mode and (iii) noise applied in multiplicative mode.

3.4. Role of dopant location (r_0):

Figure 4 depicts the variation of SPE as a function of r_0 in absence of noise [Figure 4(i)] and when noise is applied through additive [Figure 4(ii)] and multiplicative [Figure 4(iii)] modes, respectively. Under all conditions, SPE increases steadily with r_0 [58, 61] up to r_0 ~ 4.5 nm beyond which SPE nearly saturates.

The observed behavior can be explained by the fact that a gradual shift of dopant from on-center to off-center locations reduces the confinement and makes the wave function more prone to delocalization. After a typical off-center location of $r_0 \sim 4.5$ nm the extent of delocalization reaches its limit and SPE stabilizes.

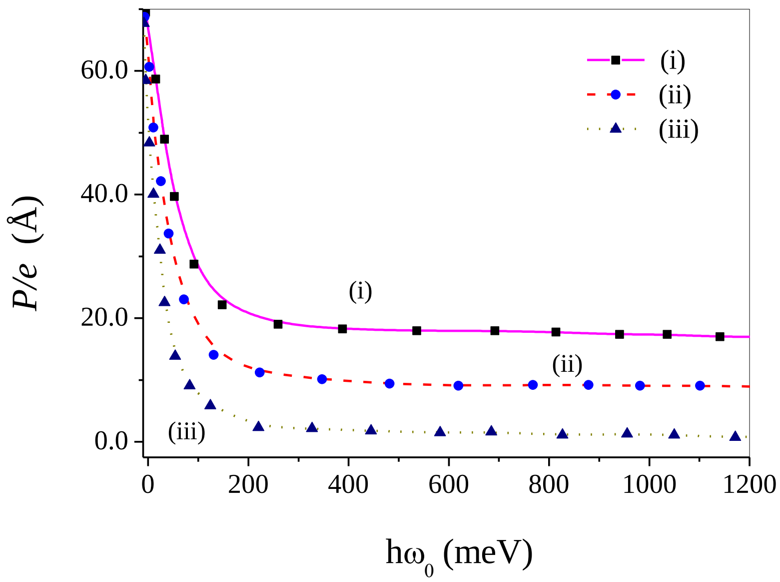

Figure 3: Plots of SPE vs *hω₀*: (i) without noise, (ii) noise applied in additive mode and (iii) noise applied in multiplicative mode.

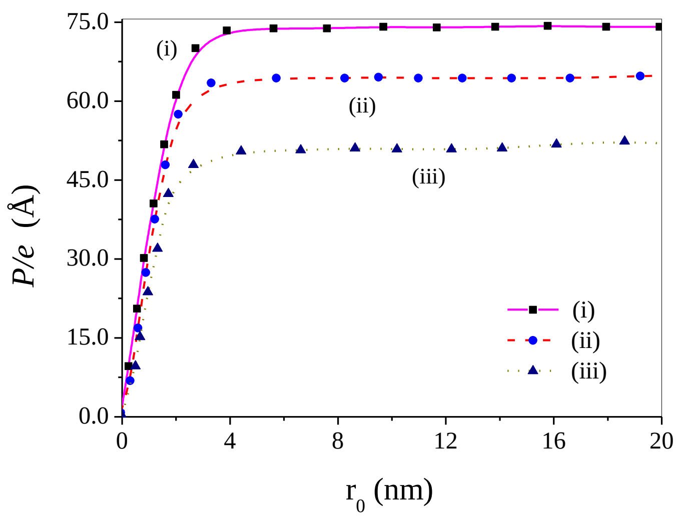

Figure 4: Plots of SPE vs r_0: (i) without noise, (ii) noise applied in additive mode and (iii) noise applied in multiplicative mode.

3.5. Role of dopant potential (V_0):

Figure 5 evinces the change of SPE with V_0 when noise is absent [Figure 5(i)] and when noise is introduced via additive [Figure 5(ii)] and multiplicative [Figure 5(iii)] pathways, respectively.

Both in absence and presence of noise SPE decreases moderately in a steady way with increase in V_0. The behavior divulges progressive compression in the spatial spread of wave function with increase in the dopant potential.

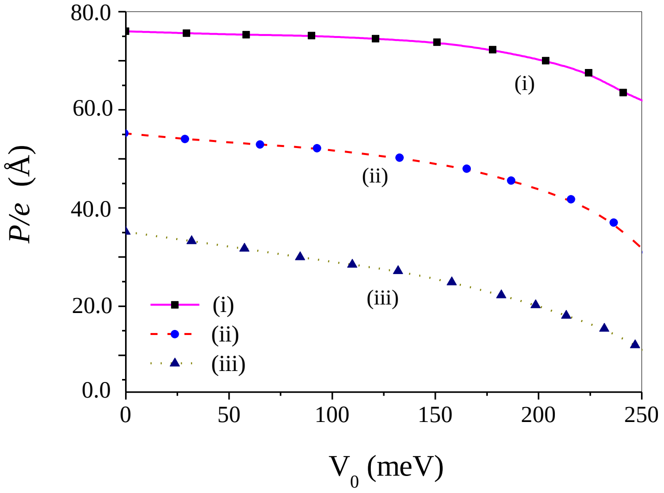

Figure 5: Plots of SPE vs V_0: (i) without noise, (ii) noise applied in additive mode and (iii) noise applied in multiplicative mode.

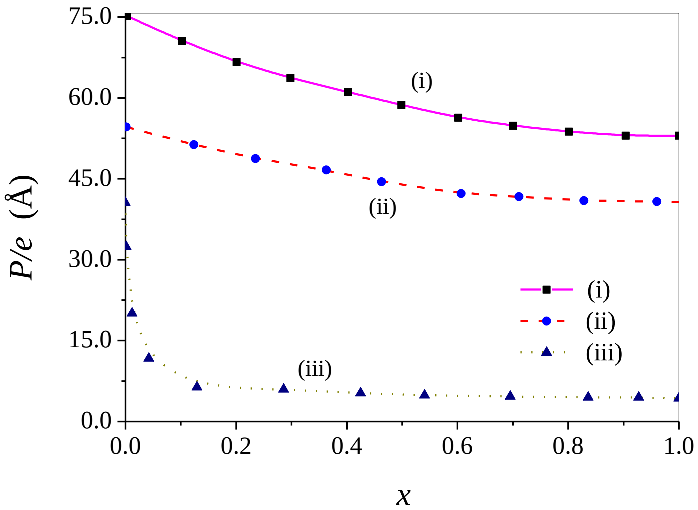

Figure 6: Plots of SPE vs *x*: (i) without noise, (ii) noise applied in additive mode and (iii) noise applied in multiplicative mode.

3.6. Role of aluminium concentration (*x*):

We now consider $Al_xGa_{1-x}As$ QD whose effective mass is given by m_*=(0.067+0.083*x*) m_0 [16]. Figure 6 evinces the change of SPE with *x* when noise is absent [Figure 6(i)] and when noise is introduced via additive [Figure 6(ii)] and multiplicative [Figure 6(iii)] pathways, respectively. Regardless of presence of noise, SPE decreases regularly with increase in aluminium concentration indicating enhanced confinement and resulting contraction of spatial stretch of wave function. However, whereas the decrease is mild in absence of noise and in presence of additive noise, it is quite sharp in presence of multiplicative noise. Thus, the spatial contraction of wave function with *x* becomes highly severe in presence of multiplicative noise.

3.7. Role of noise strength (ζ):

In the discussions made so far we have come across a common outcome that application of noise almost invariably reduces the magnitude of SPE from that of noise-free condition. It can therefore be inferred that introduction of noise diminishes the asymmetric nature of the system to a noticeable extent. Moreover, the extent of drop in the asymmetric character of the system has been found to be more with multiplicative noise than its additive analogue. Thus, the mode of application of noise also appears important in harnessing the SPE of the system.

This is because of the fact that, multiplicative noise - by virtue of its mode of application - undergoes more close connection with the system than its additive counterpart leading to greater loss of asymmetric character. It needs to be mentioned now that throughout the whole investigation the noise strength ζ has been kept equal to 1.0×10^{-2}. The question that comes in mind is that whether above observations are absolutely general or they may be different depending upon the noise strength. It therefore appears rational to compute SPE over a wide range of ζ. Figure 7 displays the plots of SPE vs $-log(\zeta)$ in presence of additive [Figure 7(i)] and multiplicative [Figure 7(ii)] noise, respectively. The plot clearly depicts that the hitherto observed sequence of SPE i.e. SPE (noise-free condition) > SPE (in presence of additive noise) > SPE (in presence of multiplicative noise) is not a general sequence but valid only when $\zeta >\sim 10^{-3}$ (the moderate-to-high noise strength regime). As ζ is lowered below $\sim 10^{-3}$, SPE in presence of multiplicative noise exceeds that in presence of additive one and even exceeds the noise-free value at $\zeta \sim 10^{-3.5}$.

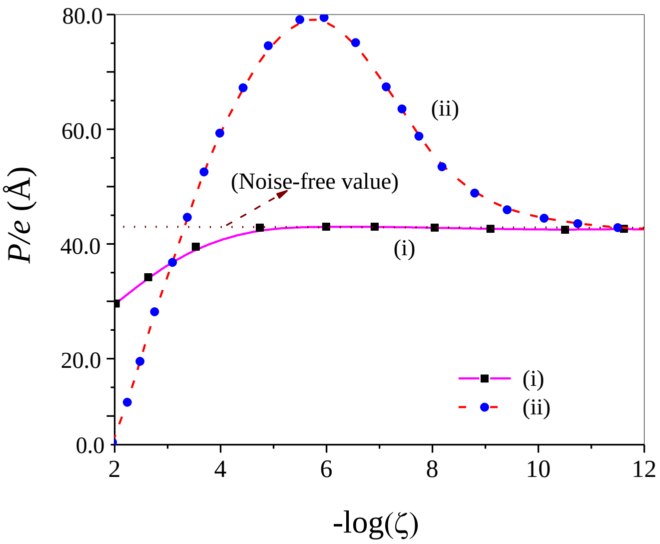

Figure 7: Plots of (a) SPE vs ζ: (i) in presence of additive noise and (ii) in presence of multiplicative noise.

Therefore, inside the noise-strength regime $\sim 10^{-3.5} < \zeta <\sim 10^{-3}$ (the moderate noise-strength regime), the sequence becomes SPE (noise-free condition) > SPE (in presence of multiplicative noise) > SPE (in presence of additive noise). SPE in presence of additive noise approaches the noise-free value at $\zeta \sim 10^{-5}$ and stabilizes. Within the low noise-strength domain i.e. $\sim 10^{-5} < \zeta <\sim 10^{-11}$, SPE in presence of multiplicative noise becomes maximum and the sequence becomes SPE (in presence of multiplicative noise) > SPE (in presence of additive noise) ~ SPE (in noise-free condition). SPE in presence of multiplicative noise reaches the noise-free value at extremely small value of noise strength ($\zeta \sim 10^{-11}$). Close scrutiny of the plot clearly reveals that noise of given strength, as well as its mode of application, play crucial role in tuning the asymmetric character of the system reflected through SPE of LDSS.

4. Conclusions

The profiles of SPE of doped *GaAs* QD have been critically analyzed in presence of noise. The profiles have been monitored against the variations of several important quantities like electric field strength, magnetic field strength, confinement potential, dopant location, dopant potential, noise strength and aluminium concentration (only for $Al_xGa_{1-x}As$ QD). Variations of these quantities, in effect, control the effective confinement of the system. The effective confinement governs the spatial elongation of wave function which is reflected through the profiles of SPE. Application of noise diminishes the magnitude of SPE from that of noise-free situation. However, the extent of reduction manifestly depends on mode of introduction of noise and also on the noise strength regime. The results seem to carry ample significance in the related field of research.

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